

Completing the square can be used to solve all quadratic equations with exact value.

Given:

$$\text{Perfect Square polynomials : } y = (ax)^2 + 2ax(b) + b^2 = (ax+b)^2$$
$$y = 4x^2 + 16x + 6$$

step 1: Move the constant.

$$y - 6 = 4x^2 + 16x$$

Step 2:

Factor out the lead coefficient.

Easier to factor when lead coefficient = 1

$$y - 6 = 4(x^2 + 4x)$$

Step 3:

a. Take half the x coefficient.

$$1/2(4) = 2$$

b. Square it.

$$2^2 = 4$$

c. Put it inside parenthesis.

$$y - 6 = 4(x^2 + 4x + 2^2)$$

d. Balance equation.

$$y - 6 + 4(4) = 4(x^2 + 4x + 2^2)$$

(Add the same amount)

Step 4:

Express the polynomial

in binomial form

$$y - 6 + 4(4) = 4(x + 2)^2$$

Step 5:

Simplify:  $y - 6 + 16 = 4(x + 2)^2$

$$y + 10 = 4(x + 2)^2$$

Step 6: Solve for roots:

$$\begin{aligned}y &= 0 \\0 + 10 &= 4(x + 2)^2 \\10/4 &= (x + 2)^2\end{aligned}$$

$$\pm \frac{\sqrt{10}}{\sqrt{4}} = x + 2$$

$$-2 \pm \frac{\sqrt{10}}{2} = x + 2 - 2$$

$$-2 \pm \frac{\sqrt{10}}{2} = x$$